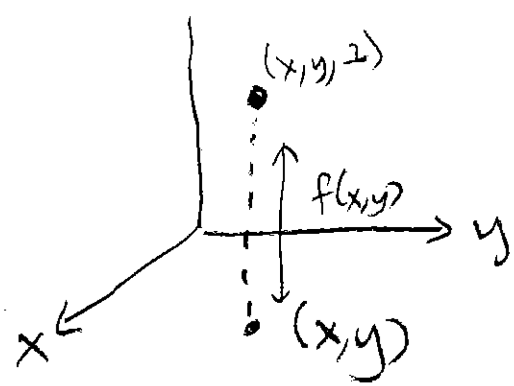
14.1/14.3 Intro to Multivariable Functions and Partial Derivatives

Def'n: A function, f, of two variables is a rule that assigns a number for each input (x,y).

$$z = f(x, y).$$

In 3D:
(x,y) is the location on the xy-plane
z = f(x,y) = height above that point.

We sometimes write $f: \mathbb{R}^2 \to \mathbb{R}$.



The set of allowable inputs is called the **domain**. Any question that asks "find the domain" is simply asking you if you know your functions well enough to understand when they are not defined.

Appears in Function	*Restriction*
$\sqrt{BLAH}$	BLAH ≥ 0
STUFF/BLAH	BLAH ≠ 0
ln(BLAH)	BLAH > 0
$\sin^{-1}$(BLAH)	-1 ≤ BLAH ≤ 1
and other trig…	

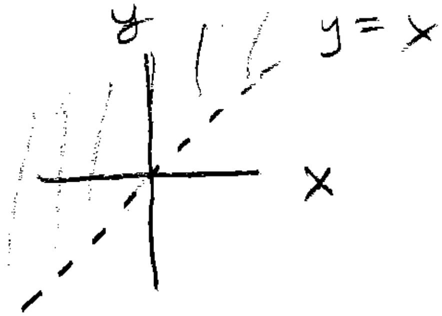
Examples:

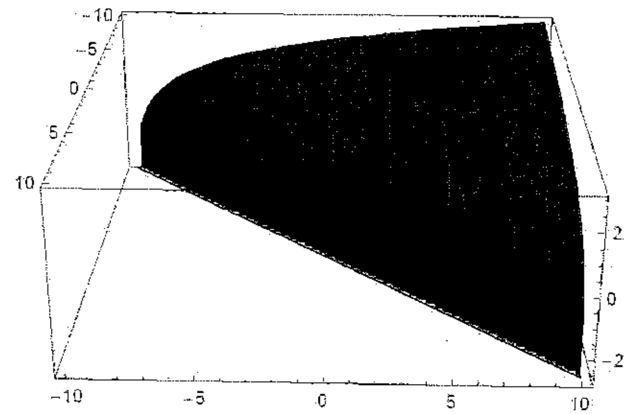
Sketch the domain of

(1) $f(x, y) = \ln(y - x)$

$y - x > 0$

$\Rightarrow \; y > x$

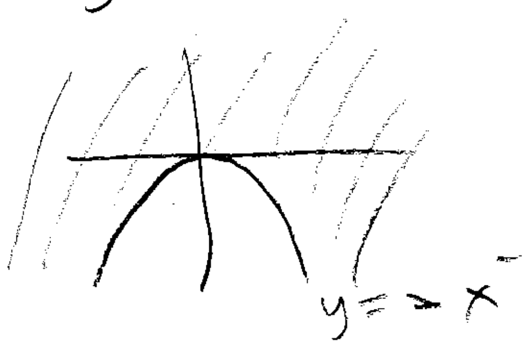


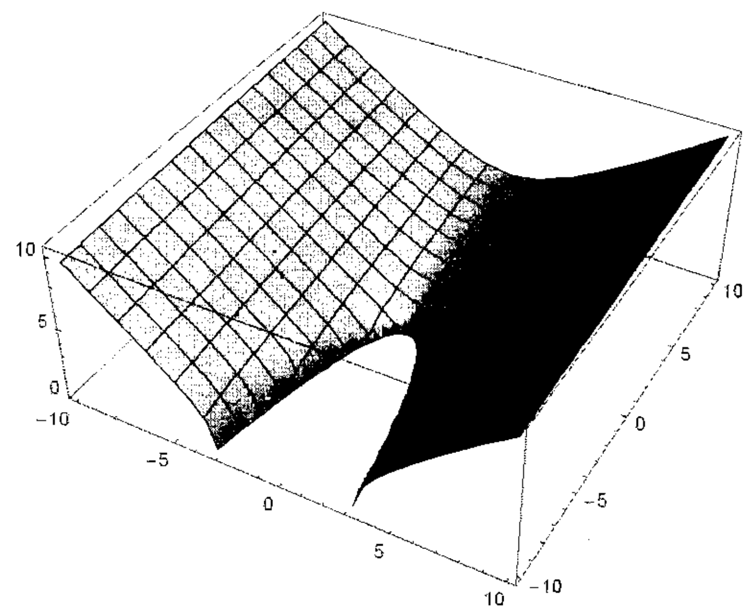


(2) $g(x, y) = \sqrt{y + x^2}$

$y + x^2 \geq 0$

$\Rightarrow \; y \geq -x^2$





Visualizing Surfaces

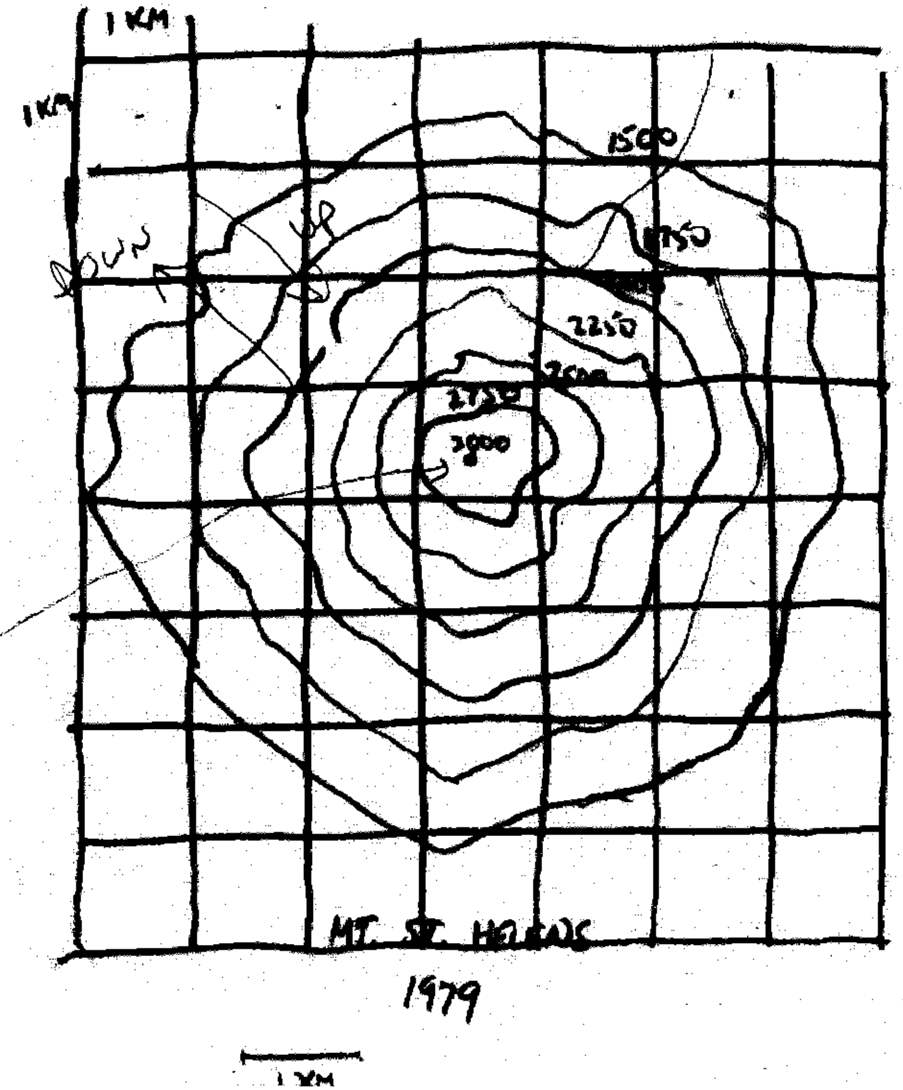
The basic tool for graphing surfaces is **traces**. We typically look at traces given by fixed values of z (height) first.

We call these traces **level curves**, because each curve represents all the points at the same height (level) on the surface. A collection of level curves is called a **contour map** (or **elevation map**).

LOCAL MAX

STEEPER IF LEVEL CURVES CLOSER TOGETHER

Contour Map (Elevation Map) of Mt. St. Helens from 1979 (before it erupted):



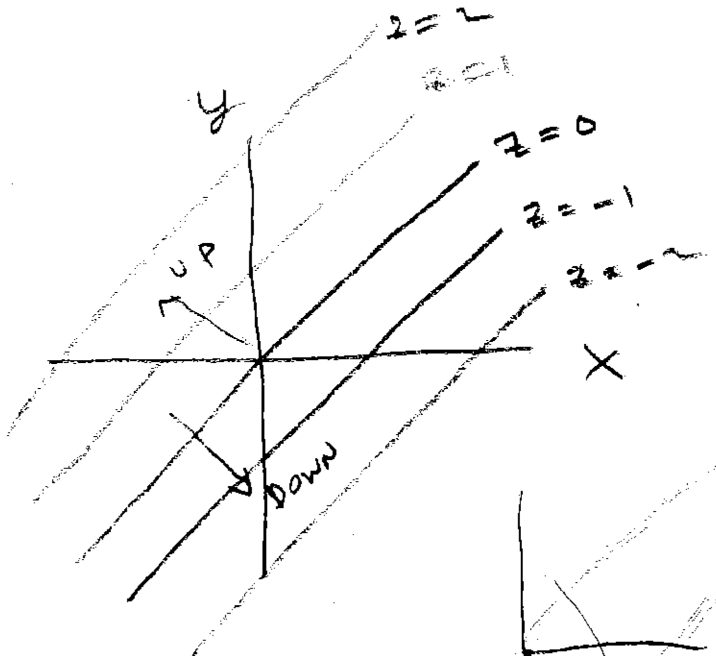
Examples:

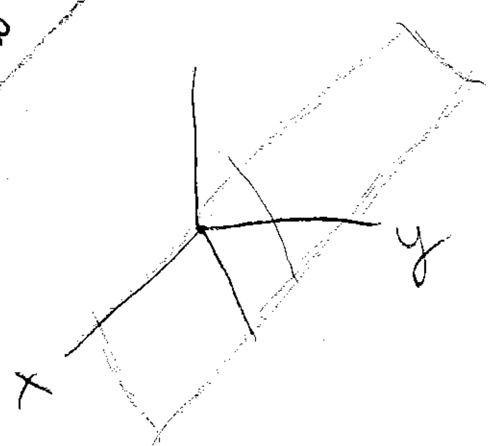
1. Graph the level curves for
   z = -2, -1, 0, 1, and 2 for
   $$z = f(x, y) = y - x$$

$-2 = y - x \iff y = x - 2$

$-1 = y - x \iff y = x - 1$

$0 = y - x \iff y = x$





PLANE!

$z = y - x$

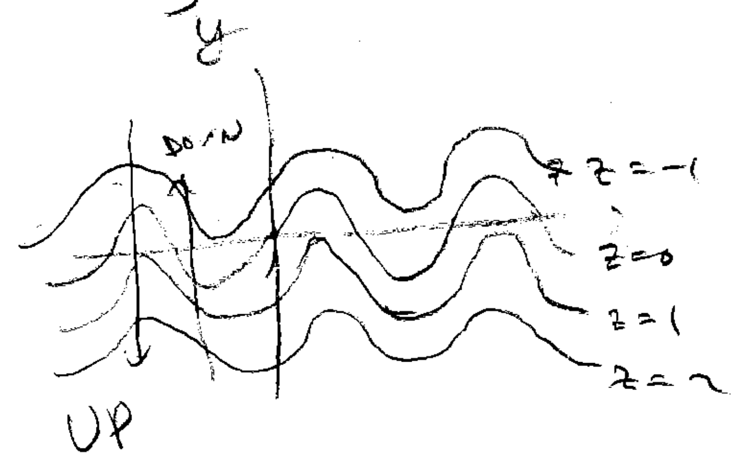
$\boxed{x - y + z = 0}$

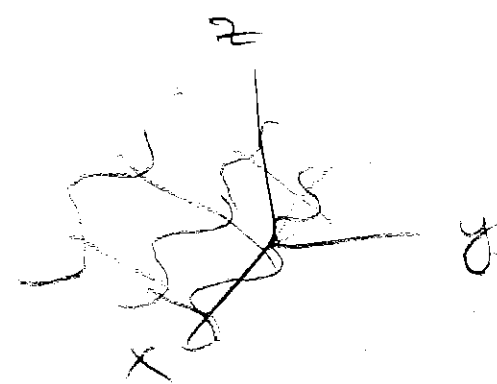
2. Graph level curves for
   $$z = f(x, y) = \sin(x) - y$$

$0 = \sin(x) - y \Rightarrow y = \sin(x)$

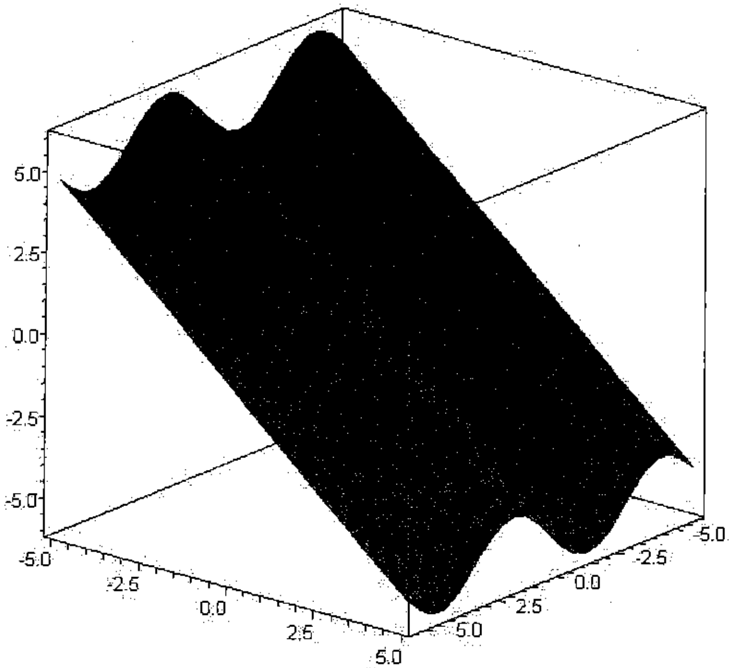
$1 = \sin(x) - y \Rightarrow y = \sin(x) - 1$

$2 = \sin(x) - y \Rightarrow y = \sin(x) - 2$





Graph of $z = \sin(x) - y$



3. Graph level curves for

$$z = f(x,y) = \frac{1}{1+x^2+y^2}$$

$z=0 \Rightarrow 0 = \frac{1}{1+x^2+y^2} \Rightarrow 0 = 1$??! NO PTS

$z=1 \Rightarrow 1 = \frac{1}{1+x^2+y^2} \Rightarrow 1+x^2+y^2 = 1 \Rightarrow x^2+y^2 = 0 \Rightarrow (x,y)=(0,0)$

$z=2 \Rightarrow 2 = \frac{1}{1+x^2+y^2} \Rightarrow 1+x^2+y^2 = \frac{1}{2} \Rightarrow x^2+y^2 = -\frac{1}{2}$ NO PTS!

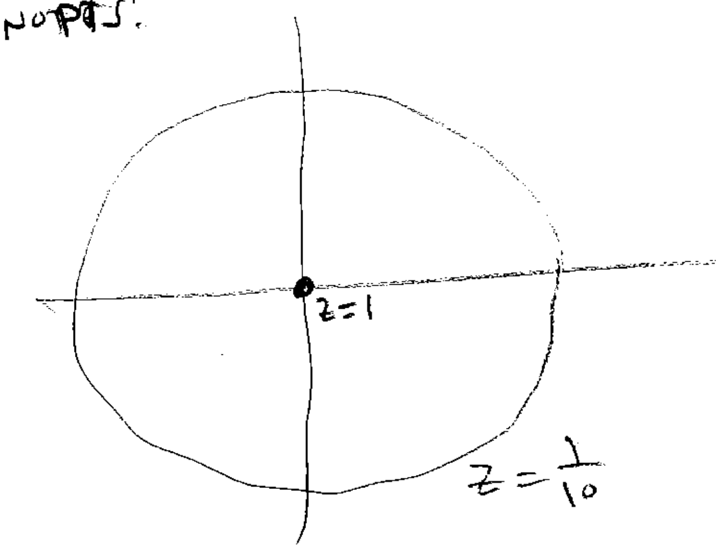
NEED TO PICK z SUCH THAT $0 < z < 1$

$z = \frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \ldots$

$$\frac{1}{10} = \frac{1}{1+x^2+y^2}$$

$$1+x^2+y^2 = 10$$

$$x^2+y^2 = 9$$



Level Curves for

$$z = f(x,y) = \frac{1}{1 + x^2 + y^2}$$

at z = 1/10, 2/10, … , 9/10, 10/10

Graph of $z = f(x,y) = \frac{1}{1+x^2+y^2}$